Balanced Three-Phase Circuits

CHAPTER 13

The generation, transmission, distribution, and utilization of large blocks of electric power are accomplished by means of three-phase circuits. The comprehensive analysis of three-phase systems is a field of study in its own right, which we cannot hope to cover in a single chapter. Fortunately, an understanding of only the steady-state sinusoidal behavior of *balanced* three-phase circuits is quite sufficient for engineers who do not specialize in power systems. We will define what we mean by a balanced circuit later in our discussion. For the moment, we note that there are two reasons for our restricting our introduction to balanced operation. First, for economic reasons, three-phase systems are designed to operate in the balanced state. This means that under normal operating conditions the three-phase circuit is so close to being balanced that we are justified in finding the solution that assumes perfect balance. Second, some types of unbalanced operating conditions can be solved by a technique known as the *method of symmetrical components*, which relies heavily on a thorough understanding of balanced operation. Although we will not discuss the method of symmetrical components, it is worth noting that an understanding of balanced operation is a starting point for an advanced technique used to analyze certain types of unbalanced conditions.

SPICE Using SPICE for sinusoidal steady-state analysis: Secs. 7 and 10 (manual pp. 27 and 43)

The basic structure of a three-phase system consists of voltage sources connected to loads via transformers[†] and transmission

[†]Transformers are introduced in Chapter 14.

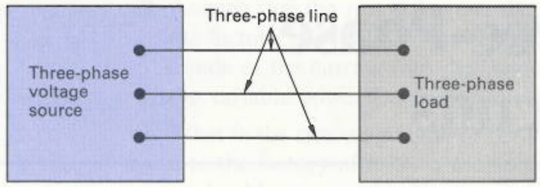


Figure 13.1 A basic three-phase circuit.

lines. We can reduce the problem to the analysis of a circuit consisting of a voltage source connected to a load via a line. The omission of the transformer as an element in the system simplifies the discussion without jeopardizing a basic understanding of the calculations involved. The basic circuit is shown in Fig. 13.1. In order to begin analyzing a circuit of this type, we must understand the characteristics of a balanced three-phase set of sinusoidal voltages.

13.1 BALANCED THREE-PHASE VOLTAGES

A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequency but are out of phase with each other by exactly 120°. In discussing three-phase circuits, it is standard practice to refer to the three phases as a, b, and c. Furthermore, the a-phase is almost always used as the reference phase. The three voltages that compose the three-phase set are referred to as the *a-phase voltage,* the *b-phase voltage,* and the *c-phase voltage*.

Phase sequence defined

Since the phase voltages are out of phase by exactly 120°, there are two possible phase relationships that can exist between the a-phase voltage and the b- and c-phase voltages. One possibility is for the b-phase voltage to lag the a-phase voltage by 120°, in which case the c-phase voltage must lead the a-phase voltage by 120°. This phase relationship is known as the *abc,* or *positive, phase sequence*. The only other possibility is for the b-phase voltage to lead the a-phase voltage by 120°, in which case the c-phase voltage must lag the a-phase voltage by 120°. This phase relationship is known as the *acb,* or *negative, phase sequence*. In phasor notation, the two possible sets of balanced phase voltages are

Balanced phase voltages (positive sequence)

$$\mathbf{V}_a = V_m\underline{/0^\circ},$$
$$\mathbf{V}_b = V_m\underline{/-120^\circ}, \qquad (13.1)$$
$$\mathbf{V}_c = V_m\underline{/+120^\circ},$$

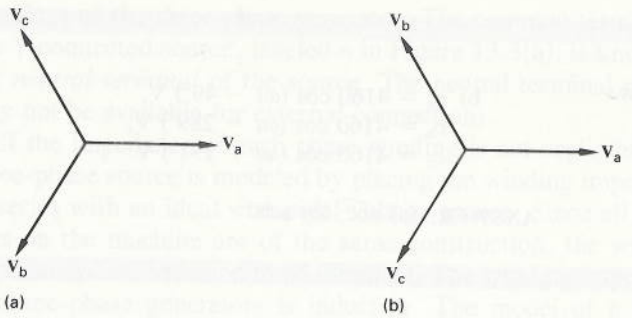


Figure 13.2 Phasor diagrams of a balanced set of three-phase voltages: (a) abc (positive) sequence and (b) acb (negative) sequence.

and

$$\mathbf{V}_a = V_m\angle 0^\circ,$$
$$\mathbf{V}_b = V_m\angle +120^\circ, \quad (13.2)$$
$$\mathbf{V}_c = V_m\angle -120^\circ.$$

Balanced phase voltages (negative sequence)

The phase sequence of the voltages given by Eqs. (13.1) is the abc, or positive, sequence. The phase sequence of the voltages given by Eqs. (13.2) is the acb, or negative, sequence. The phasor diagram representations of the voltage sets given by Eqs. (13.1) and (13.2) are shown in Fig. 13.2, from which we can determine the phase sequence by noting the order of the subscripts as we move clockwise around the diagram. The fact that a three-phase circuit can have one of two possible phase sequences is an important characteristic that must be taken into account whenever two separate circuits are operated in parallel. The two circuits can operate in parallel only if they have the same phase sequence.

Another important characteristic of a set of balanced three-phase voltages is that the sum of the voltages adds to zero. Thus using either Eqs. (13.1) or Eqs. (13.2) we have

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0. \quad (13.3)$$

Sum of a balanced set is zero

The fact that the sum of the phasor voltages adds to zero also means that the sum of the instantaneous voltages is zero, that is,

$$v_a + v_b + v_c = 0. \quad (13.4)$$

Another noteworthy observation is that if we know the *phase sequence and one voltage in the set, we know the entire set*. Thus if we have a balanced three-phase system, we can focus on determining the voltage (or current) in one phase, because once we know one phase quantity we automatically know the corresponding quantity in the other two phases.

One voltage plus the sequence defines a set

DRILL EXERCISES

13.1 What is the phase sequence of each of the following sets of voltages?

a) $v_a = 208 \cos (\omega t + 76°)$ V,
$v_b = 208 \cos (\omega t + 316°)$ V,
$v_c = 208 \cos (\omega t - 164°)$ V.

b) $v_a = 4160 \cos (\omega t - 49°)$ V,
$v_b = 4160 \cos (\omega t - 289°)$ V,
$v_c = 4160 \cos (\omega t + 191°)$ V.

ANSWER: (a) abc; (b) acb.

13.2 THREE-PHASE VOLTAGE SOURCES

Three-phase voltage sources consist of generators that have three separate windings distributed around the periphery of the stator. Each winding composes one phase of the generator. The rotor of the generator is an electromagnet driven at synchronous speed by a prime mover such as a steam or gas turbine. As the electromagent is rotated past the three windings, a sinusoidal voltage is induced in each winding. The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by exactly 120°. Since the phase windings are stationary with respect to the rotating electromagent, the frequency of the voltage induced in each winding is the same.

Ideal model of a three-phase generator

Normally, the impedance of each phase winding on a three-phase generator is very small compared with the other impedances in the circuit. Therefore, to an approximation, each phase winding can be modeled in an electric circuit by an ideal sinusoidal voltage source. There are two ways of interconnecting the separate phase windings to form a three-phase source. The windings can be connected together in either a wye (Y) or a delta (Δ) configuration. The wye and delta connections are shown in Fig. 13.3, where ideal voltage sources are used to model the phase

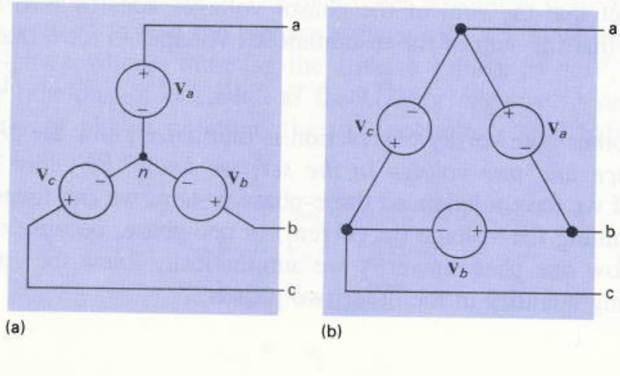


Figure 13.3 The two basic connections of an ideal three-phase source: (a) Y-connected source and (b) Δ-connected source.

windings of the three-phase generator. The common terminal in the Y-connected source, labeled n in Figure 13.3(a), is known as the *neutral terminal* of the source. The neutral terminal may or may not be available for external connections.

Neutral terminal

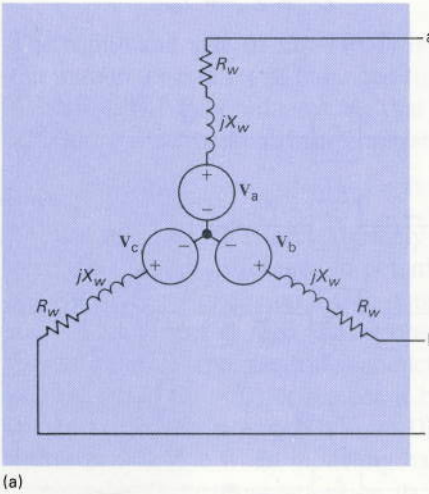
If the impedance of each phase winding is not negligible, the three-phase source is modeled by placing the winding impedance in series with an ideal sinusoidal voltage source. Since all windings on the machine are of the same construction, the winding impedances are assumed to be identical. The winding impedance of three-phase generators is inductive. The model of a three-phase source including winding impedance is shown in Fig. 13.4, in which R_w is the winding resistance and X_w is the inductive reactance of the winding.

Practical model

The fact that a three-phase voltage source can be either a Y-connected or Δ-connected means that the basic circuit in Fig. 13.1 can take four different configurations, since the three-phase loads can also be either Y-connected or Δ-connected. The four possible arrangements are (1) a Y-connected source and a Y-connected load; (2) a Y-connected source and Δ-connected load; (3) a Δ-connected source and a Y-connected load; and (4) a Δ-connected source and a Δ-connected load.

Four possible configurations for three-phase circuits

We begin our analysis of three-phase circuits with the first arrangement mentioned above. After analyzing the Y–Y circuit, we will show for balanced circuits how the remaining three arrangements can be reduced to a Y–Y equivalent circuit. In other words, the analysis of the Y–Y circuit is the key to solving all balanced three-phase arrangements.



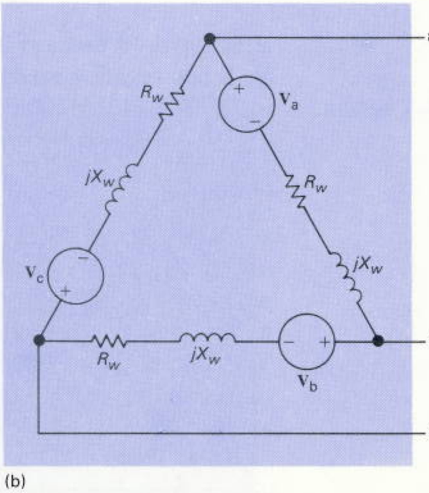


Figure 13.4 A model of a three-phase source with winding impedance: (a) Y-connected source and (b) Δ-connected source.

13.3 ANALYSIS OF THE WYE–WYE CIRCUIT

We begin our analysis of the Y–Y circuit by assuming that the circuit is *not* balanced! We do this so that we can show what we mean by a balanced three-phase circuit and what the consequences of being balanced are in terms of circuit analysis. The general Y–Y circuit is illustrated in Fig. 13.5, where we have included a fourth conductor that connects the source neutral to the load neutral. The fourth conductor is possible only in the Y–Y arrangement. (More about this later.) We also mention that for convenience in drawing the diagram, we have transformed the Y-connections into "tipped-over tees." In Fig. 13.5, Z_{ga}, Z_{gb}, and Z_{gc} represent the internal impedance associated with each phase winding of the voltage source; Z_{1a}, Z_{1b}, and Z_{1c} represent the impedance of each phase conductor of the line connecting the source to the load; Z_o is the impedance of the neutral conductor that connects the source neutral to the load neutral; and Z_A, Z_B, and Z_C represent the impedance of each phase of the load.

The circuit in Fig. 13.5 can be described by a single node-voltage equation. Using the source neutral as the reference node and letting $\mathbf{V}_N$ denote the node voltage between the nodes N and n, we find that the node-voltage equation is

Nodal equation for the circuit in Fig. 13.5

$$\frac{\mathbf{V}_N}{Z_o} + \frac{\mathbf{V}_N - \mathbf{V}_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{\mathbf{V}_N - \mathbf{V}_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{\mathbf{V}_N - \mathbf{V}_{c'n}}{Z_C + Z_{1c} + Z_{gc}} = 0. \quad (13.5)$$

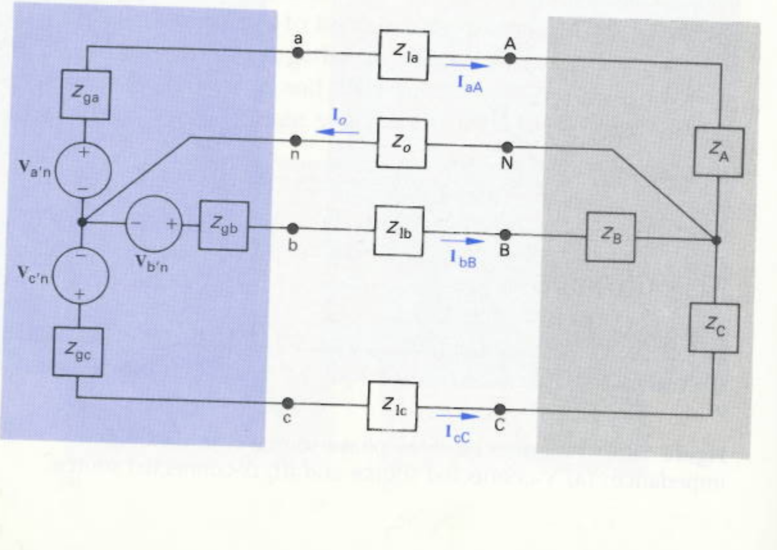


Figure 13.5 A three-phase Y–Y system.

Before pursuing Eq. (13.5) any further, let us pause to observe that the circuit analysis techniques that we have discussed in the earlier chapters are directly applicable to three-phase circuits. Thus it is not necessary to introduce new analytical techniques in order to analyze three-phase circuits. However, as we will see in the remainder of this chapter, if a three-phase circuit is balanced we can take some significant analytical shortcuts to predict the behavior of the system.

The circuit in Fig. 13.5 is a balanced three-phase circuit if it satisfies *all* of the following criteria:

Definition of a balanced three-phase circuit

1. $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, and $\mathbf{V}_{c'n}$ form a set of balanced three-phase voltages,
2. $Z_{ga} = Z_{gb} = Z_{gc}$,
3. $Z_{1a} = Z_{1b} = Z_{1c}$,
4. $Z_A = Z_B = Z_C$.

There is no restriction on the impedance of the neutral conductor (Z_o); its value has no effect on whether or not the system is balanced.

If the system is balanced, Eq. (13.5) tells us that $\mathbf{V}_N$ must be zero. To see this let

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} \tag{13.6}$$

and then rewrite Eq. (13.5) as

$$\mathbf{V}_N\left(\frac{1}{Z_o} + \frac{3}{Z_\phi}\right) = \frac{\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n}}{Z_\phi}. \tag{13.7}$$

The right-hand side of Eq. (13.7) is zero because by hypothesis the numerator is a set of balanced three-phase voltages and Z_ϕ is *not* zero. The only value of $\mathbf{V}_N$ that satisfies Eq. (13.7) is zero. Therefore for a balanced three-phase circuit,

$$\mathbf{V}_N = 0. \tag{13.8}$$

Equation (13.8) is an extremely important result. If $\mathbf{V}_N$ is zero, there is no difference in potential between the source neutral (n) and the load neutral (N); consequently, the current in the neutral conductor is zero. These observations tell us that we can either remove the neutral conductor from a balanced Y–Y configuration ($I_0 = 0$) or replace it by a perfect short circuit between the nodes n and N ($\mathbf{V}_N = 0$). We find both equivalents convenient to use when modeling balanced three-phase circuits.

Now let us turn our attention to what effect balanced conditions have on the three line currents. It follows directly from

Fig. 13.5 that when the system is balanced, the three line currents will be

Line currents in a balanced Y-Y circuit

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{\mathbf{V}_{a'n}}{Z_\phi}, \quad (13.9)$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{\mathbf{V}_{b'n}}{Z_\phi}, \quad (13.10)$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_c + Z_{1c} + Z_{gc}} = \frac{\mathbf{V}_{c'n}}{Z_\phi}, \quad (13.11)$$

from which we see that in a balanced system the three line currents form a balanced set of three-phase currents. Thus the current in each line will be equal in amplitude and frequency and will be exactly 120° out of phase with the other two line currents. This tells us that if we calculate the current $\mathbf{I}_{aA}$, we can write down the line currents $\mathbf{I}_{bB}$ and $\mathbf{I}_{cC}$ without further computations. We imply by this statement that the phase sequence is known.

We can use Eq. (13.9) to construct a single-phase equivalent circuit of the balanced three-phase Y–Y circuit. It follows from Eq. (13.9) that the current in the a-phase conductor line is simply the voltage generated in the a-phase winding of the generator divided by the total impedance in the a-phase of the circuit. Thus Eq. (13.9) describes the simple circuit in Fig. 13.6, where the neutral conductor has been replaced by a perfect short circuit. A word of caution here. The current in the neutral conductor of Fig. 13.6 is *not* the current in the neutral conductor of a balanced three-phase circuit. The current in the neutral conductor is

$$\mathbf{I}_o = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC}, \quad (13.12)$$

whereas the current in the neutral conductor in Fig. 13.6 is $\mathbf{I}_{aA}$. Thus the circuit in Fig. 13.6 gives the correct value of the line current but only the a-phase component of the neutral current. Whenever the single-phase equivalent circuit in Fig. 13.6 is applicable, the line currents form a balanced three-phase set and the right-hand side of Eq. (13.12) sums to zero.

Once we know the line currents in the circuit in Fig. 13.5, it is a relatively simple task to calculate any voltages that are of interest. In particular, we are interested in the relationship between the line-to-line voltages and the line-to-neutral voltages. We will establish this relationship at the load terminals. The observations we make will also apply at the source terminals. The line-to-line voltages at the terminals of the load in terms of the line-to-neutral voltages at the load are

Line-to-line voltages

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}, \quad (13.13)$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}, \quad (13.14)$$

and

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}. \qquad \textbf{(13.15)}$$

The double-subscript notation in voltage equations indicates that the voltage is a drop from the first subscript to the second subscript. The relationships given by Eqs. (13.13) through (13.15) are shown in Fig. 13.7. Because we are interested in the balanced state, we have omitted the neutral conductor from the figure.

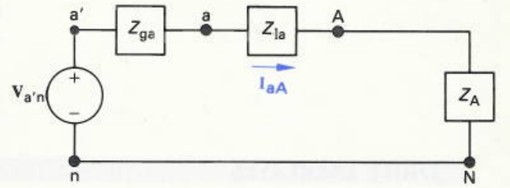


Figure 13.6 A single-phase equivalent circuit.

To show the relationship between the line-to-line voltages and the line-to-neutral voltages, we assume a positive, or abc, sequence. We arbitrarily choose the line-to-neutral voltage of the a-phase as the reference. Thus,

$$\mathbf{V}_{AN} = V_\phi \underline{/0^\circ}, \qquad \textbf{(13.16)}$$

$$\mathbf{V}_{BN} = V_\phi \underline{/-120^\circ}, \qquad \textbf{(13.17)}$$

and

$$\mathbf{V}_{CN} = V_\phi \underline{/+120^\circ}, \qquad \textbf{(13.18)}$$

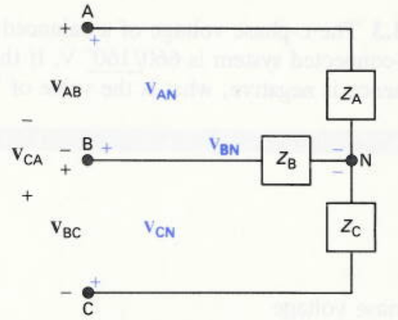


Figure 13.7 Line-to-line and line-to-neutral voltages.

where V_ϕ represents the magnitude of the line-to-neutral voltage. When we substitute Eqs. (13.16) through (13.18) into Eqs. (13.13) through (13.15), respectively, we get

$$\mathbf{V}_{AB} = V_\phi - V_\phi \underline{/-120^\circ} = \sqrt{3}\, V_\phi \underline{/30^\circ}, \qquad \textbf{(13.19)}$$

$$\mathbf{V}_{BC} = V_\phi \underline{/-120^\circ} - V_\phi \underline{/120^\circ} = \sqrt{3}\, V_\phi \underline{/-90^\circ}, \qquad \textbf{(13.20)}$$

and

$$\mathbf{V}_{CA} = V_\phi \underline{/120^\circ} - V_\phi \underline{/0^\circ} = \sqrt{3}\, V_\phi \underline{/150^\circ}. \qquad \textbf{(13.21)}$$

Line and phase voltages in a Y-connected load

From Eqs. (13.19) through (13.21), we see that (1) the magnitude of the line-to-line voltage is $\sqrt{3}$ times the magnitude of the line-to-neutral voltage, (2) the line-to-line voltages form a balanced three-phase set of voltages, and (3) the set of line-to-line voltages lead the set of line-to-neutral voltages by 30°. We will leave it to the reader to show that for a negative, or acb, sequence the only change is that the set of line-to-line voltages lags the set of line-to-neutral voltages by 30°. These observations are summarized in the phasor diagrams of Fig. 13.8. We

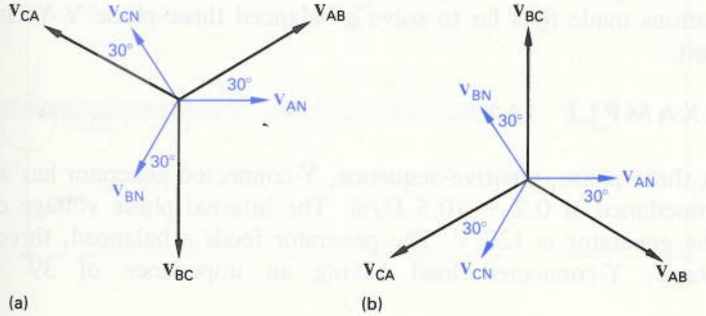


Figure 13.8 Phasor diagrams showing the relationship between line-to-line and line-to-neutral voltages in a balanced system: (a) abc sequence and (b) acb sequence.

now observe that, in a balanced system, if the line-to-neutral voltage is known at some point in the circuit, the line-to-line voltages at the same point are also known, and vice versa.

DRILL EXERCISES

13.2 The voltage from B to N in a balanced three-phase circuit is $120\underline{/60^\circ}$ V. If the phase sequence is positive, what is the value of V_{BC}?

ANSWER: $207.85\underline{/+90^\circ}$ V.

13.3 The c-phase voltage of a balanced, three-phase, Y-connected system is $660\underline{/160^\circ}$ V. If the phase sequence is negative, what is the value of V_{AB}?

ANSWER: $1143.15\underline{/-110^\circ}$ V.

Phase voltage
Line voltage
Phase current
Line current

Before illustrating balanced three-phase calculations with a numerical example, we must make some additional comments on terminology. In the Y–Y system, the line-to-neutral voltage is also called the *phase voltage*. For brevity, the line-to-line voltage is also called the *line voltage*. The *phase current* is defined as the current in each phase of the load or, at the source end of the circuit, the current in each phase of the generator. The *line current* is defined as the current in each phase of the line. For the Y–Y arrangement, the phase current and line current are identical. Because three-phase systems are designed to handle large blocks of electric power, all voltage and current specifications and calculations are given in terms of rms values. Thus when a three-phase transmission line is rated at 345 kV, this means that the nominal value of the rms line-to-line voltage is 345,000 V. *All voltages and currents in this chapter are understood to be rms values*. Finally, the Greek letter phi (ϕ) is widely used in the literature to denote a per-phase quantity. Thus $\mathbf{V}_\phi$, $\mathbf{I}_\phi$, Z_ϕ, P_ϕ, and Q_ϕ are interpreted as voltage/phase, current/phase, impedance/phase, power/phase, and reactive power/phase, respectively.

Example 13.1 is designed to show how we can use the observations made thus far to solve a balanced three-phase Y–Y circuit.

EXAMPLE 13.1

Solving a balanced three-phase Y-Y circuit

A three-phase, positive-sequence, Y-connected generator has an impedance of $0.2 + j0.5\ \Omega/\phi$. The internal phase voltage of the generator is 120 V. The generator feeds a balanced, three-phase, Y-connected load having an impedance of 39 +

$j28\ \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8 + j1.5\ \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Construct a single-phase equivalent circuit of the three-phase system.

b) Calculate the three line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the three line-to-neutral voltages at the load: $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, $\mathbf{V}_{CN}$.

d) Calculate the line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ at the terminals of the load.

e) Calculate the line-to-neutral voltages at the terminals of the generator $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$.

f) Calculate the line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ at the terminals of the generator.

g) Repeat parts (a) through (f), given that the phase sequence is negative.

SOLUTION

a) The single-phase equivalent circuit is shown in Fig. 13.9.

b) The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120\angle 0^\circ}{40 + j30} = 2.4\angle -36.87^\circ\ \text{A}.$$

For a positive phase sequence,

$$\mathbf{I}_{bB} = 2.4\angle -156.87^\circ\ \text{A},$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^\circ\ \text{A}.$$

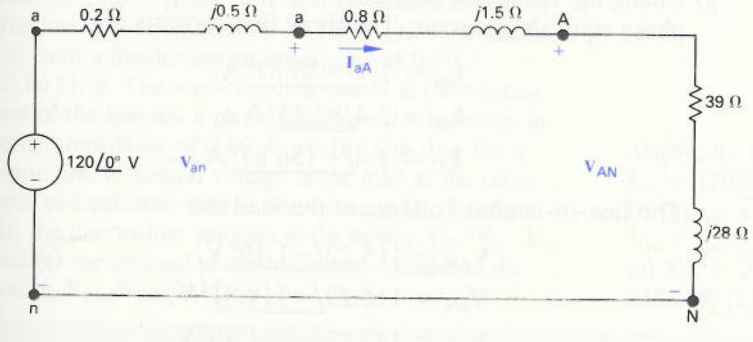


Figure 13.9 The single-phase equivalent circuit for Example 13.1.

c) The line-to-neutral voltage at the A terminal of the load is

$$\mathbf{V}_{AN} = (39 + j28)(2.4\underline{/-36.87^\circ})$$
$$= 115.22\underline{/-1.19^\circ}\text{ V}.$$

For a positive phase sequence,

$$\mathbf{V}_{BN} = 115.22\underline{/-121.19^\circ}\text{ V},$$
$$\mathbf{V}_{CN} = 115.22\underline{/+118.81^\circ}\text{ V}.$$

d) For a positive phase sequence, the line-to-line voltages lead the line-to-neutral voltages by 30°; thus

$$\mathbf{V}_{AB} = (\sqrt{3}\underline{/30^\circ})\,\mathbf{V}_{AN}$$
$$= 199.58\underline{/28.81^\circ}\text{ V},$$
$$\mathbf{V}_{BC} = 199.58\underline{/-91.19^\circ}\text{ V},$$
$$\mathbf{V}_{CA} = 199.58\underline{/148.81^\circ}\text{ V}.$$

e) The line-to-neutral voltage at the a-terminal of the source is

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4\underline{/-36.87^\circ})$$
$$= 120 - 1.29\underline{/31.33^\circ}$$
$$= 118.90 - j0.67$$
$$= 118.90\underline{/-0.32^\circ}\text{ V}.$$

For a positive phase sequence,

$$\mathbf{V}_{bn} = 118.90\underline{/-120.32^\circ},$$
$$\mathbf{V}_{cn} = 118.90\underline{/119.68^\circ}\text{ V}.$$

f) The line-to-line voltages at the source terminals are

$$\mathbf{V}_{ab} = (\sqrt{3}\underline{/30^\circ})\,\mathbf{V}_{an}$$
$$= 205.94\underline{/29.68^\circ}\text{ V},$$
$$\mathbf{V}_{bc} = 205.94\underline{/-90.32^\circ}\text{ V}$$
$$\mathbf{V}_{ca} = 205.94\underline{/149.68^\circ}\text{ V}.$$

g) Changing the phase sequence has no effect on the single-phase equivalent circuit. The three line currents are

$$\mathbf{I}_{aA} = 2.4\underline{/-36.87^\circ}\text{ A},$$
$$\mathbf{I}_{bB} = 2.4\underline{/83.13^\circ}\text{ A},$$
$$\mathbf{I}_{cC} = 2.4\underline{/-156.87^\circ}\text{ A}.$$

The line-to-neutral voltages at the load are

$$\mathbf{V}_{AN} = 115.22\underline{/-1.19^\circ}\text{ V},$$
$$\mathbf{V}_{BN} = 115.22\underline{/+118.81^\circ}\text{ V},$$
$$\mathbf{V}_{CN} = 115.22\underline{/-121.19^\circ}\text{ V}.$$

For a negative phase sequence, the line-to-line voltages lag the line-to-neutral voltages by 30°:

$$\mathbf{V}_{AB} = (\sqrt{3}\angle{-30^\circ})\mathbf{V}_{AN}$$
$$= 199.58\angle{-31.19^\circ}\text{ V},$$
$$\mathbf{V}_{BC} = 199.58\angle{88.81^\circ}\text{ V},$$
$$\mathbf{V}_{CA} = 199.58\angle{-151.19^\circ}\text{ V}.$$

The line-to-neutral voltages at the terminals of the generator are

$$\mathbf{V}_{an} = 118.90\angle{-0.32^\circ}\text{ V},$$
$$\mathbf{V}_{bn} = 118.90\angle{119.68^\circ}\text{ V},$$
$$\mathbf{V}_{cn} = 118.90\angle{-120.32^\circ}\text{ V}.$$

The line-to-line voltages at the terminals of the generator are

$$\mathbf{V}_{ab} = (\sqrt{3}\angle{-30^\circ})\mathbf{V}_{an}$$
$$= 205.94\angle{-30.32^\circ}\text{ V},$$
$$\mathbf{V}_{bc} = 205.94\angle{89.68^\circ}\text{ V},$$
$$\mathbf{V}_{ca} = 205.94\angle{-150.32^\circ}\text{ V}.$$

In studying Example 13.1, it is important to note that once the a-phase quantity is calculated, the corresponding b- and c-phase values can be tabulated by simply shifting the a-phase value by 120°. For a positive phase sequence, the b-phase lags the a-phase by 120°, whereas the c-phase leads the a-phase by 120°. For a negative phase sequence, the b-phase leads the a-phase by 120° and the c-phase lags the a-phase by 120°. We also call your attention to how easy it is to calculate line-to-line voltages once we know the line-to-neutral voltages.

DRILL EXERCISES

13.4 The line-to-neutral voltage at the terminals of a balanced, three-phase, wye-connected load is 2400 V. The load has an impedance of $16 + j12\ \Omega/\phi$ and is fed from a line having an impedance of $0.10 + j0.80\ \Omega/\phi$. The wye-conected source at the sending end of the line has a phase sequence of acb and an internal impedance of $0.02 + j0.16\ \Omega/\phi$. Use the a-phase line-to-neutral voltage at the load as the reference and calculate (a) the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, $\mathbf{I}_{cC}$; (b) the line-to-line voltages at the source $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$; and (c) the internal phase-to-neutral voltages at the source $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, $\mathbf{V}_{c'n}$.

ANSWER: (a) $\mathbf{I}_{aA} = 120\angle{-36.87^\circ}$ A, $\mathbf{I}_{bB} = 120\angle{83.13^\circ}$ A, $\mathbf{I}_{cC} = 120\angle{-156.87^\circ}$ A; (b) $\mathbf{V}_{ab} = 4275.02\angle{-28.38^\circ}$ V, $\mathbf{V}_{bc} = 4275.02\angle{91.62^\circ}$ V, $\mathbf{V}_{ca} = 4275.02\angle{-148.38^\circ}$ V; (c) $\mathbf{V}_{a'n} = 2482.05\angle{1.93^\circ}$ V, $\mathbf{V}_{b'n} = 2482.05\angle{121.93^\circ}$ V, $\mathbf{V}_{c'n} = 2482.05\angle{-118.07^\circ}$ V.

13.4 ANALYSIS OF THE WYE–DELTA CIRCUIT

If the load in a three-phase circuit is connected in a delta, it can be transformed into a wye by using the delta-to-wye transformation discussed in Section 11.6. When the load is balanced, the impedance of each leg of the wye is one-third the impedance of each leg of the delta; thus

$$Z_Y = \frac{Z_\Delta}{3}, \tag{13.22}$$

which follows directly from Eq. (13.47) through (13.49). Once the Δ-load has been replaced by its Y-equivalent, the Y-source, Δ-load, three-phase circuit can be modeled by the single-phase equivalent circuit in Fig. 13.6.

After we have used the single-phase equivalent circuit to calculate the line currents, we can find the current in each leg of the original Δ-load by simply dividing the line currents by $\sqrt{3}$ and shifting them through 30°. This relationship between the line currents and phase currents in the delta can be derived using the circuit in Fig. 13.10.

When a load, or source, is connected in a delta, the current in each leg of the delta is the phase current and the voltage across each leg is the phase voltage. We can see from Fig. 13.10 that in the Δ-configuration, the phase voltage is identical with the line voltage.

To see the relationship between the phase currents and line currents, we will assume a positive phase sequence and let I_ϕ represent the magnitude of the phase current. It follows, then, that

$$\mathbf{I}_{AB} = I_\phi \angle 0°, \tag{13.23}$$

$$\mathbf{I}_{BC} = I_\phi \angle -120°, \tag{13.24}$$

Figure 13.10 A circuit used to establish the relationship between line currents and phase currents in a balanced delta load.

and

$$\mathbf{I}_{CA} = I_\phi \angle +120°. \tag{13.25}$$

In writing these equations, we have arbitrarily selected $\mathbf{I}_{AB}$ as the reference phasor.

We can write the line currents in terms of the phase currents by direct application of Kirchhoff's current law; thus

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = I_\phi \angle 0° - I_\phi \angle 120° = \sqrt{3} I_\phi \angle -30°, \tag{13.26}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = I_\phi \angle -120° - I_\phi \angle 0° = \sqrt{3} I_\phi \angle -150°, \tag{13.27}$$

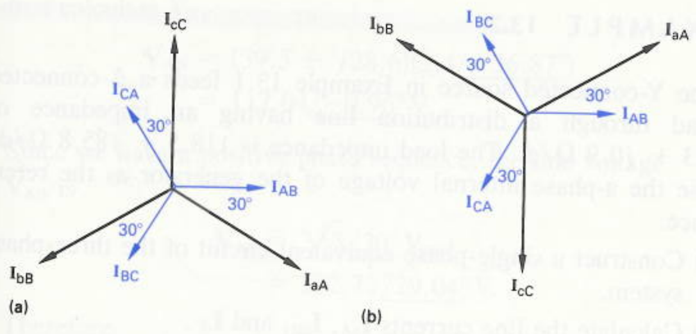


Figure 13.11 Phasor diagrams showing the relationship between line currents and phase currents in a Δ-connected load: (a) positive sequence and (b) negative sequence.

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = I_\phi \angle 120^\circ - I_\phi \angle -120^\circ$$
$$= \sqrt{3} I_\phi \angle 90^\circ. \quad (13.28)$$

When we compare Eqs. (13.26) through (13.28) with (Eqs. 13.23) through (13.25), we see that the magnitude of the line currents is $\sqrt{3}$ times the magnitude of the phase currents and the set of line currents *lags* the set of phase currents by 30°.

We will leave it to the reader to verify that for a negative phase sequence, the line currents are $\sqrt{3}$ times larger than the phase currents and *lead* the phase currents by 30°.

The relationship between the line currents and the phase currents of a Δ-connected load are summarized in Fig. 13.11.

Line and phase currents in a Δ-connected load

DRILL EXERCISES

13.5 The current $\mathbf{I}_{CA}$ in a balanced, three-phase, Δ-connected load is $15\angle 38^\circ$A. If the phase sequence is positive, what is the value of $\mathbf{I}_{cC}$?

ANSWER: $25.98\angle 8^\circ$A.

13.6 A balanced, three-phase, Δ-connected load is fed from a balanced three-phase circuit. The reference for the b-phase line current is toward the load. The value of the current is $26\angle -50^\circ$A. If the phase sequence is negative, what is the value of $\mathbf{I}_{AB}$?

ANSWER: $15.01\angle 160^\circ$A.

Example 13.2 illustrates the calculations involved in analyzing a balanced three-phase circuit involving a Y-connected source and a Δ-connected load.

EXAMPLE 13.2

Analysis of the Y-Δ circuit

The Y-connected source in Example 13.1 feeds a Δ-connected load through a distribution line having an impedance of $0.3 + j0.9\ \Omega/\phi$. The load impedance is $118.5 + j85.8\ \Omega/\phi$. Use the a-phase internal voltage of the generator as the reference.

a) Construct a single-phase equivalent circuit of the three-phase system.

b) Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the phase voltages at the terminals of the load.

d) Calculate the phase currents of the load.

e) Calculate the line voltages at the terminals of the source.

SOLUTION

a) The single-phase equivalent circuit is shown in Fig. 13.12. The load impedance of the Y-equivalent is $(1/3)(118.5 + j85.8)$, or $39.5 + j28.6\ \Omega/\phi$.

b) The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\underline{/0^\circ}}{(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6)}$$

$$= \frac{120\underline{/0^\circ}}{40 + j30} = 2.4\underline{/-36.87^\circ}\text{A}.$$

It follows directly that

$$\mathbf{I}_{bB} = 2.4\underline{/-156.87^\circ}\text{ A}$$

and

$$\mathbf{I}_{cC} = 2.4\underline{/83.13^\circ}\text{A}.$$

c) Since the load is Δ-connected, the phase voltages are the same as the line voltages. To calculate the line voltages, we

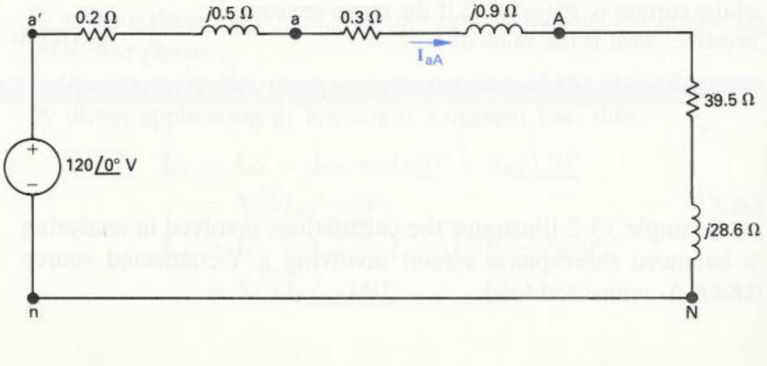


Figure 13.12 The single-phase equivalent circuit for Example 13.2.

first calculate $\mathbf{V}_{AN}$:

$$\mathbf{V}_{AN} = (39.5 + j28.6)(2.4\underline{/-36.87^\circ}) = 117.04\underline{/-0.96^\circ}\text{ V}.$$

Since we have a positive phase sequence, the line voltage $\mathbf{V}_{AB}$ is

$$\mathbf{V}_{AB} = \sqrt{3}\underline{/30^\circ}\,\mathbf{V}_{AN} = 202.72\underline{/29.04^\circ}\text{ V}.$$

Therefore

$$\mathbf{V}_{BC} = 202.72\underline{/-90.96^\circ}\text{ V}$$

and

$$\mathbf{V}_{CA} = 202.72\underline{/149.04^\circ}\text{ V}.$$

d) The phase currents of the load can be calculated directly from the line currents. Then

$$\mathbf{I}_{AB} = \frac{1}{\sqrt{3}}\underline{/30^\circ}\,\mathbf{I}_{aA} = 1.39\underline{/-6.87^\circ}\text{ A}.$$

Once we know $\mathbf{I}_{AB}$, we also know the other load phase currents:

$$\mathbf{I}_{BC} = 1.39\underline{/-126.87^\circ}\text{ A}$$

and

$$\mathbf{I}_{CA} = 1.39\underline{/113.13^\circ}\text{ A}.$$

Note that we can check our calculation of $\mathbf{I}_{AB}$ using the previously calculated $\mathbf{V}_{AB}$ and the impedance of the Δ-connected load. That is,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\phi} = \frac{202.72\underline{/29.04^\circ}}{118.5 + j85.8} = 1.39\underline{/-6.87^\circ}\text{ A}.$$

(Alternative methods of calculation are very helpful in eliminating errors and are highly recommended in all work involving analysis and design.)

e) To calculate the line voltage at the terminals of the source, we first calculate $\mathbf{V}_{an}$. We see from Fig. 13.12 that $\mathbf{V}_{an}$ is the voltage drop across the line impedance plus the load impedance. Thus

$$\mathbf{V}_{an} = (39.8 + j29.5)2.4\underline{/-36.87^\circ} = 118.90\underline{/-0.32^\circ}\text{ V}.$$

The line voltage $\mathbf{V}_{ab}$ is

$$\mathbf{V}_{ab} = \sqrt{3}\,\underline{/30^\circ}\,\mathbf{V}_{an}$$

or

$$\mathbf{V}_{ab} = 205.94\,\underline{/29.68^\circ}\text{ V}.$$

Therefore

$$\mathbf{V}_{bc} = 205.94\,\underline{/-90.32^\circ}\text{ V},$$
$$\mathbf{V}_{ca} = 205.94\,\underline{/+149.68^\circ}\text{ V}.$$

DRILL EXERCISES

13.7 The line-to-line voltage $\mathbf{V}_{AB}$ at the terminals of a balanced, three-phase, Δ-connected load is $4160\underline{/0^\circ}$ V. The line current $\mathbf{I}_{aA}$ is $69.28\underline{/-10^\circ}$ A.

a) Calculate the per-phase impedance of the load if the phase sequence is positive.

b) Repeat part (a) if the phase sequence is negative.

ANSWER: (a) $104\underline{/-20^\circ}\ \Omega$; (b) $104\underline{/+40^\circ}\ \Omega$.

13.8 The line voltage at the terminals of a balanced, Δ-connected load is 208 V. Each phase of the load consists of a 5.2-Ω resistor in parallel with a 6.933-Ω inductor. What is the magnitude of the current in the line feeding the load?

ANSWER: 86.60 A.

13.5 ANALYSIS OF THE DELTA–WYE CIRCUIT

In the Δ–Y three-phase circuit, the source is Δ-connected and the load is Y-connected. We can obtain the single-phase equivalent circuit by replacing the balanced Δ-connected source by a Y-equivalent. We can obtain the Y-equivalent of the source by dividing the internal phase voltages of the Δ-source by $\sqrt{3}$ and shifting this set of three-phase voltages by -30° if the phase sequence is positive and by $+30^\circ$ if the phase sequence is negative. The internal impedance of the Y-equivalent is one-third the internal impedance of the Δ-source. The Y-equivalent circuit of a positive-sequence Δ-connected source is illustrated in Fig. 13.13.

For a positive phase sequence, the set of Δ-source phase currents ($\mathbf{I}_{ba}$, $\mathbf{I}_{cb}$, and $\mathbf{I}_{ac}$ in Fig. 13.13) lead the set of line currents

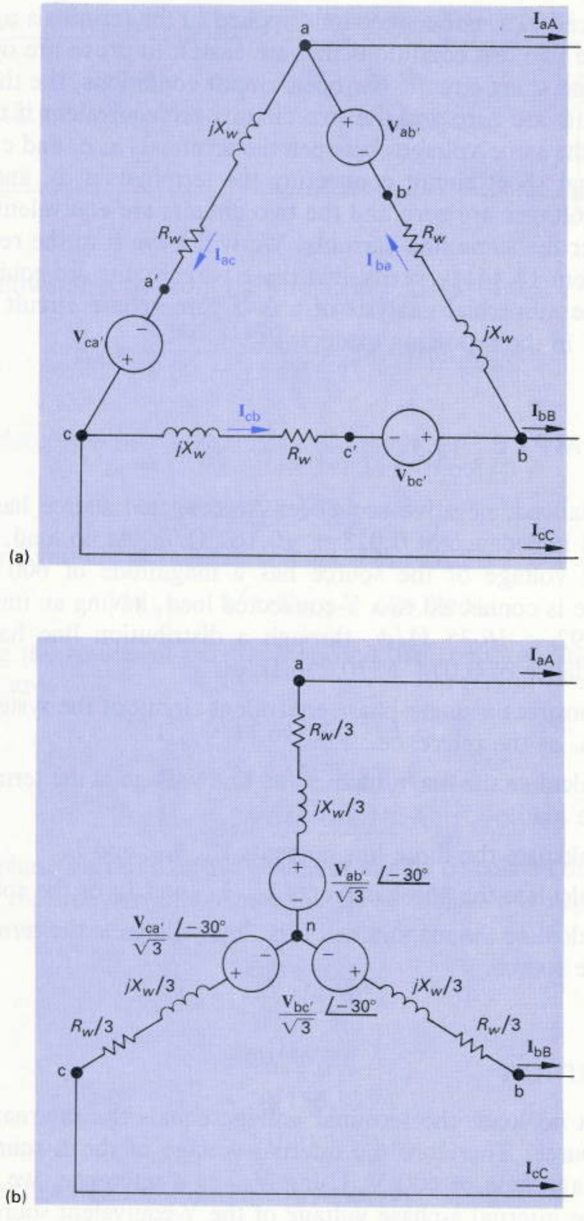


Figure 13.13 The Y-equivalent of a balanced, three-phase, Δ-connected source (positive sequence): (a) Δ source and (b) Y-equivalent.

$\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ by 30°. The magnitude of the phase currents is $1/\sqrt{3}$ times the magnitude of the line currents. For a negative phase sequence, the phase currents in the source lag the line currents by 30°.

Y equivalent of a Δ source

To show that the Y-source of Fig. 13.13(b) is equivalent to the Δ-source of Fig. 13.13(a), it is necessary to show only that the two circuits produce the same terminal conditions for any balanced *external* connections applied to the terminals a, b, and c. The two test conditions that are easiest to prove are open circuit and short circuit. For open-circuit conditions, the three line currents are zero and the two circuits are equivalent if they deliver the same voltages between the terminals a, b, and c. For an external short circuit connecting the terminals a, b, and c, the line voltages are zero and the two circuits are equivalent if they deliver the same line currents. We will leave it to the reader (in Problem 13.14) to verify that these two circuits are equivalent.

The numerical analysis of a Δ–Y three-phase circuit is illustrated in the following example.

EXAMPLE 13.3

Analysis of a Δ-Y three-phase circuit

A balanced, negative-sequence, Δ-connected source has an internal impedance of $0.018 + j0.162\ \Omega/\phi$. At no load, the terminal voltage of the source has a magnitude of 600 V. The source is connected to a Y-connected load, having an impedance of $7.92 - j6.35\ \Omega/\phi$, through a distribution line having an impedance of $0.074 + j0.296\ \Omega/\phi$.

a) Construct a single-phase equivalent circuit of the system using $\mathbf{V}_{ab'}$ as the reference.

b) Calculate the magnitude of the line voltage at the terminals of the load.

c) Calculate the three line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

d) Calculate the phase currents $\mathbf{I}_{ba}$, $\mathbf{I}_{cb}$, and $\mathbf{I}_{ac}$ of the source.

e) Calculate the magnitude of the line voltage at the terminals of the source.

SOLUTION

a) At no load, the terminal voltage equals the internal voltage source. Therefore the internal voltage of the Δ-source has a magnitude of 600 V. Using $\mathbf{V}_{ab'}$ as a reference, we find that the internal a-phase voltage of the Y-equivalent source is

$$\mathbf{V}_{a'n} = \frac{\mathbf{V}_{ab'}}{\sqrt{3}}\underline{/30^\circ} = \frac{600}{\sqrt{3}}\underline{/30^\circ}$$

$$\cong 346.41\underline{/30^\circ}\text{V}.$$

The internal impedance of the equivalent Y-generator is $(1/3)(0.018 + j0.162)$, or $0.006 + j0.054\ \Omega/\phi$. Therefore the single-phase equivalent circuit is as shown in Fig. 13.14.

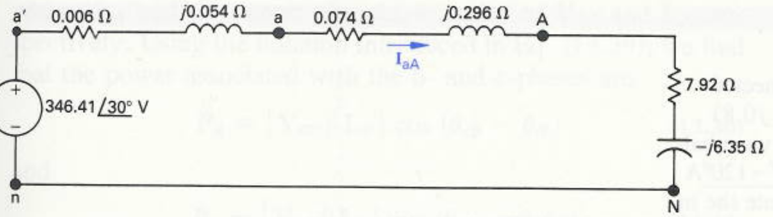


Figure 13.14 The single-phase equivalent circuit for Example 13.3.

b) It follows directly from the circuit in Fig. 13.14 that

$$\mathbf{I}_{aA} = \frac{346.41\underline{/30^\circ}}{8.00 - j6.00} = 34.64\underline{/66.87^\circ}\text{ A}$$

and

$$\mathbf{V}_{AN} = (7.92 - j6.35)(34.64\underline{/66.87^\circ})$$
$$= 351.65\underline{/28.15^\circ}\text{V}.$$

The magnitude of the line voltage at the load is

$$|\mathbf{V}_{AB}| = \sqrt{3}\,|\mathbf{V}_{AN}| = 609.08\text{ V}.$$

c) Using the results of part (b), we find that the three line currents are

$$\mathbf{I}_{aA} = 34.64\underline{/66.87^\circ}\text{ A},$$
$$\mathbf{I}_{bB} = 34.64\underline{/186.87^\circ}\text{ A},$$
$$\mathbf{I}_{cC} = 34.64\underline{/-53.13^\circ}\text{ A}.$$

d) The phase currents of the generator can be calculated directly from the line currents. Since the phase sequence is negative, we have

$$\mathbf{I}_{ba} = \frac{1}{\sqrt{3}}\underline{/-30^\circ}\,\mathbf{I}_{aA}$$
$$= 20\underline{/36.87^\circ}\text{ A},$$
$$\mathbf{I}_{cb} = 20\underline{/156.87^\circ}\text{ A},$$
$$\mathbf{I}_{ac} = 20\underline{/-83.13^\circ}\text{ A}.$$

e) From the circuit in Fig. 13.14, we have

$$\mathbf{V}_{an} = (7.994 - j6.054)\mathbf{I}_{aA}$$
$$= 34.64(7.994 - j6.054)\underline{/66.87^\circ}$$
$$= 347.37\underline{/29.73^\circ}\text{ V}.$$

The magnitude of the line voltage at the source will be

$$|\mathbf{V}_{ab}| = \sqrt{3}\,|\mathbf{V}_{an}| = 601.66\text{ V}.$$

DRILL EXERCISES

13.9 A balanced, positive-sequence , Δ-connected source has an internal impedance of $0.09 + j0.81$ Ω/ϕ. The source is feeding a balanced load via a balanced line. The b-phase line current $\mathbf{I}_{bB}$ is $6\underline{/-120^\circ}$A and the line voltage $\mathbf{V}_{ab}$ is $480\underline{/60^\circ}$ V. Calculate the internal source voltage $\mathbf{V}_{ab'}$.

ANSWER: $481.68\underline{/60.27^\circ}$ V.

13.6 ANALYSIS OF THE DELTA-DELTA CIRCUIT

In the Δ–Δ circuit, both the source and the load are Δ-connected. The single-phase equivalent circuit of a balanced Δ–Δ system is obtained by replacing both the source and the load with their Y-equivalents. As before, the Y-equivalent circuit is used to solve for line currents and line-to-neutral voltages. Once we know the line currents, we can find the phase currents in both the load and the source using the techniques described in Sections 13.4 and 13.5. The line-to-neutral voltages can be converted to line-to-line voltages as described in Section 13.3. All these techniques have been illustrated in Examples 13.1, 13.2, and 13.3. You can gain additional experience with these types of calculations by solving Problems 13.12 through 13.17.

13.7 POWER CALCULATIONS IN BALANCED THREE-PHASE CIRCUITS

Thus far, our analysis of balanced three-phase circuits has been limited to the determination of currents and voltages in a given circuit. We are now ready to discuss three-phase power calculations. We begin by discussing the average power delivered to a balanced, Y-connected load.

Average Power in a Balanced Y-Load

A Y-connected load, along with pertinent currents and voltages, is shown in Fig. 13.15. The average power associated with any one phase can be calculated using the techniques introduced in Chapter 12. Using Eq. (12.27) as a starting point, we find that we can express the average power associated with the a-phase of the load as

$$P_A = |\mathbf{V}_{AN}||\mathbf{I}_{aA}|\cos(\theta_{vA} - \theta_{iA}), \qquad (13.29)$$

where θ_{vA} and θ_{iA} denote the phase angles of $\mathbf{V}_{AN}$ and $\mathbf{I}_{aA}$, respectively. Using the notation introduced in Eq. (13.29), we find that the power associated with the b- and c-phases are

$$P_B = |\mathbf{V}_{BN}||\mathbf{I}_{bB}| \cos(\theta_{vB} - \theta_{iB}) \quad (13.30)$$

and

$$P_C = |\mathbf{V}_{CN}||\mathbf{I}_{cC}| \cos(\theta_{vC} - \theta_{iC}). \quad (13.31)$$

In Eqs. (13.29) through (13.31), all phasor currents and voltages are written in terms of the rms value of the sinusoidal function that they represent.

In a balanced three-phase system, the magnitude of each line-to-neutral voltage is the same, as is the magnitude of each phase current. The argument of the cosine functions is also the same for all three phases. To emphasize these observations, we introduce the following notation to facilitate further discussion of power calculations in balanced three-phase circuits:

$$V_\phi = |\mathbf{V}_{AN}| = |\mathbf{V}_{BN}| = |\mathbf{V}_{CN}|, \quad (13.32)$$

$$I_\phi = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|, \quad (13.33)$$

and

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}. \quad (13.34)$$

We also note from the above observations that for a balanced system the power delivered to each phase of the load is the same; thus

Average power per phase

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi, \quad (13.35)$$

where P_ϕ stands for average power per phase.

The total average power delivered to the balanced Y-connected load is simply three times the power per phase; thus

Total average power

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi. \quad (13.36)$$

It is also desirable to express the total power in terms of the *rms magnitude* of the line voltage and the *rms magnitude* of the line current. If we let V_L represent the rms magnitude of the line voltage and I_L represent the rms magnitude of the line current, then we can modify Eq. (13.36) as follows:

Total average power in terms of line voltage and current

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi$$
$$= \sqrt{3}\, V_L I_L \cos\theta_\phi. \quad (13.37)$$

In deriving Eq. (13.37), we have used the fact that for a balanced Y-connected load, the magnitude of the phase voltage is the magnitude of the line voltage divided by $\sqrt{3}$ and the magnitude of the line current is equal to the magnitude of the phase

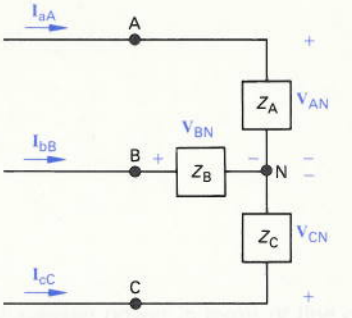


Figure 13.15 A balanced Y load used to introduce average power calculations in three-phase circuits.

current. In using Eq. (13.37) to calculate the total power delivered to the load, it is important to remember that θ_ϕ is the phase angle between the *phase voltage* and the *phase current*.

Complex Power in a Balanced Y-Load

The reactive power and complex power associated with any one phase of a Y-connected load can also be calculated using the techniques introduced in Chapter 12. For the balanced load the expressions for the reactive power are

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi \quad \textbf{(13.38)}$$

and

$$Q_T = 3Q_\phi = \sqrt{3}\,V_L I_L \sin\theta_\phi. \quad \textbf{(13.39)}$$

Complex power per phase

Equation (12.33) is the basis for expressing the complex power associated with any phase. For a balanced load we have

$$S = \mathbf{V}_{AN}\mathbf{I}_{aA}^* = \mathbf{V}_{BN}\mathbf{I}_{bB}^* = \mathbf{V}_{CN}\mathbf{I}_{cC}^* = \mathbf{V}_\phi\mathbf{I}_\phi^*, \quad \textbf{(13.40)}$$

where $\mathbf{V}_\phi$ and $\mathbf{I}_\phi$ are used to represent a phase voltage and current taken from the same phase. Thus, in general,

Total complex power

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi\mathbf{I}_\phi^* \quad \textbf{(13.41)}$$

and

$$S_T = 3S_\phi = \sqrt{3}V_L I_L \underline{/\theta_\phi}. \quad \textbf{(13.42)}$$

Power Calculations in a Balanced Δ-Load

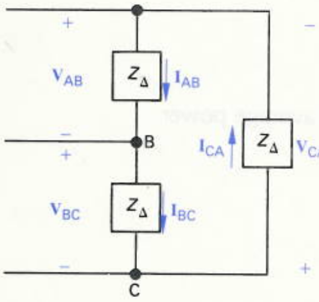


Figure 13.16 A Δ-connected load used to discuss power calculations.

If the load is Δ-connected, the calculation of power—reactive power or complex power—is basically the same as that for the Y-connected load. A Δ-connected load, along with the pertinent currents and voltages, is shown in Fig. 13.16, from which it follows that the power associated with each phase is

$$P_A = |\mathbf{V}_{AB}||\mathbf{I}_{AB}|\cos(\theta_{vAB} - \theta_{iAB}), \quad \textbf{(13.43)}$$

$$P_B = |\mathbf{V}_{BC}||\mathbf{I}_{BC}|\cos(\theta_{vBC} - \theta_{iBC}), \quad \textbf{(13.44)}$$

$$P_C = |\mathbf{V}_{CA}||\mathbf{I}_{CA}|\cos(\theta_{vCA} - \theta_{iCA}). \quad \textbf{(13.45)}$$

For a balanced load,

$$|\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_\phi, \quad \textbf{(13.46)}$$

$$|\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| = I_\phi, \quad \textbf{(13.47)}$$

$$\theta_{vAB} - \theta_{iAB} = \theta_{vBC} - \theta_{iBC} = \theta_{vCA} - \theta_{iCA} = \theta_\phi, \quad \textbf{(13.48)}$$

Average power per phase

and

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi. \quad \textbf{(13.49)}$$

It is worth noting that Eq. (13.49) is the same as Eq. (13.35). This is equivalent to saying, "In a balanced load the average power per phase is equal to the product of the rms magnitude of the phase voltage, the rms magnitude of the phase current, and the cosine of the angle between the phase voltage and phase current."

The total power delivered to a balanced Δ-connected load is

Total average power in terms of line current and voltage

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi$$
$$= 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\theta_\phi$$
$$= \sqrt{3}\,V_L I_L \cos\theta_\phi. \quad \textbf{(13.50)}$$

Note that Eq. (13.50) is the same as Eq. (13.37).

The expressions for reactive power and complex power also have the same form as those developed for the Y-load:

Complex power: per phase and total

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi, \quad \textbf{(13.51)}$$
$$Q_T = 3Q_\phi = 3V_\phi I_\phi \sin\theta_\phi, \quad \textbf{(13.52)}$$
$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*, \quad \textbf{(13.53)}$$
$$S_T = 3S_\phi = \sqrt{3}V_L I_L \underline{/\theta_\phi}. \quad \textbf{(13.54)}$$

The following examples illustrate power calculations in balanced three-phase circuits.

EXAMPLE 13.4

Power calculations: balanced three-phase circuits

a) Calculate the average power per phase delivered to the Y-connected load of Example 13.1.
b) Calculate the total average delivered by the load.
c) Calculate the total average power lost in the line.
d) Calculate the total average power lost in the generator.
e) Calculate the total number of magnetizing vars absorbed by the load.
f) Calculate the total complex power delivered by the source.

SOLUTION

a) From Example 13.1, we have $V_\phi = 115.22$ V, $I_\phi = 2.4$ A, and $\theta_\phi = -1.19 - (-36.87) = 35.68°$. Therefore

$$P_\phi = (115.22)(2.4)\cos 35.68°$$
$$= 224.64 \text{ W}.$$

We also note that the power per phase can be calculated from $I_\phi^2 R_\phi$, or

$$P_\phi = (2.4)^2(39) = 224.64 \text{ W}.$$

b) The total average power delivered to the load is $P_T = 3P_\phi = 673.92$ W. Since the line voltage was calculated in Example 13.1, we could also use Eq. (13.37); thus

$$P_T = \sqrt{3}(199.58)(2.4)\cos 35.68^\circ$$
$$= 673.92 \text{ W}.$$

c) The total power lost in the line is

$$P_{\text{line}} = 3(2.4)^2(0.8) = 13.824 \text{ W}.$$

d) The total internal power loss in the generator is

$$P_{\text{gen}} = 3(2.4)^2(0.2) = 3.456 \text{ W}.$$

e) The total number of magnetizing vars absorbed by the load is

$$Q_T = \sqrt{3}(199.58)(2.4)\sin 35.68^\circ$$
$$= 483.84 \text{ VAR}.$$

f) The total complex power associated with the source is

$$S_T = 3S_\phi = -3(120)(2.4)\underline{/36.87^\circ}$$
$$= -691.20 - j518.40 \text{ VA}.$$

The minus sign tells us that the internal power and magnetizing reactive power are being delivered to the circuit. We can check this result by calculating the total power and reactive power absorbed by the circuit. Thus

$$P = 673.92 + 13.824 + 3.456$$
$$= 691.20 \text{ W} \quad \text{(check)};$$
$$Q = 483.84 + 3(2.4)^2(1.5) + 3(2.4)^2(0.5)$$
$$= 483.84 + 25.92 + 8.64$$
$$= 518.40 \text{ VAR} \quad \text{(check)}.$$

EXAMPLE 13.5

Complex power in a balanced three-phase circuit

a) Calculate the total complex power delivered to the Δ-connected load of Example 13.2.

b) What percentage of the average power at the sending end of the line is delivered to the load?

SOLUTION

a) Using the a-phase values from the solution of Example 13.2, we have

$$\mathbf{V}_\phi = \mathbf{V}_{AB} = 202.72\underline{/29.04^\circ} \text{ V},$$
$$\mathbf{I}_\phi = \mathbf{I}_{AB} = 1.39\underline{/-6.87^\circ} \text{ A}.$$

Using Eqs. (13.53) and (13.54) we have

$$S_T = 3(202.72\underline{/29.04^\circ})(1.39\underline{/+6.87^\circ})$$
$$= 682.56 + j494.208 \text{ VA}.$$

b) The total power at the sending end of the distribution line will be equal to the total power delivered to the load plus the total power lost in the line; therefore

$$P_{\text{input}} = 682.56 + 3(2.4)^2(0.3)$$
$$= 687.744 \text{ W}.$$

The percentage of the average power at the input of the line reaching the load is 682.56/687.744, or 99.25%.

EXAMPLE 13.6

Calculations in a balanced three-phase circuit

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8. The load is fed from a line having an impedance of $0.005 + j0.025\ \Omega/\phi$. The line voltage at the terminals of the load is 600 V.

a) Construct a single-phase equivalent circuit of the system.
b) Calculate the magnitude of the line current.
c) Calculate the magnitude of the line voltage at the sending end of the line.
d) Calculate the power factor at the sending end of the line.

SOLUTION

a) The single-phase equivalent circuit is shown in Fig. 13.17. We have arbitrarily selected the line-to-neutral voltage at the load as the reference.

b) The line current $\mathbf{I}_{aA}$ is

$$\left(\frac{600}{\sqrt{3}}\right)\mathbf{I}_{aA}^* = (160 + j120)10^3$$

or

$$\mathbf{I}_{aA}^* = 577.35\underline{/36.87^\circ} \text{ A}$$

Therefore, $\mathbf{I}_{aA} = 577.35\underline{/-36.87^\circ}$ A. The magnitude of the line current is the magnitude of $\mathbf{I}_{aA}$:

$$I_L = 577.35 \text{ A}.$$

An alternative solution for I_L is obtained from the expression

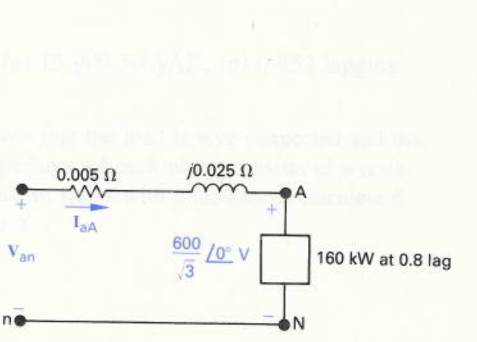


Figure 13.17 The single-phase equivalent circuit for Example 13.6.

$$P_T = \sqrt{3}V_L I_L \cos\theta_p$$
$$= \sqrt{3}(600)I_L(0.8) = 480{,}000 \text{ W};$$
$$I_L = \frac{480{,}000}{\sqrt{3}(600)(0.8)} = \frac{1000}{\sqrt{3}} = 577.35 \text{ A}.$$

c) To calculate the magnitude of the line voltage at the sending end of the line, we first calculate $\mathbf{V}_{an}$. It follows directly from Fig. 13.17 that

$$\mathbf{V}_{an} = \mathbf{V}_{an} + Z_l \mathbf{I}_{aA}$$
$$= \frac{600}{\sqrt{3}} + (0.005 + j0.025)(577.35\angle{-36.87^\circ})$$
$$= 357.51\angle{1.57^\circ}\text{V}.$$

Thus

$$V_L = \sqrt{3}|\mathbf{V}_{an}| = 619.23 \text{ V}.$$

d) The power factor at the sending end of the line is the cosine of the phase angle between $\mathbf{V}_{an}$ and $\mathbf{I}_{aA}$:

$$\text{pf} = \cos[1.57^\circ - (-36.87^\circ)]$$
$$= \cos 38.44^\circ = 0.783 \text{ lagging}.$$

An alternative method for calculating the power factor is to first calculate the complex power at the sending end of the line:

$$S_\phi = (160 + j120)10^3 + (577.35)^2(0.005 + j0.025)$$
$$= 161.67 + j128.33 \text{ kVA}$$
$$= 206.41\angle{38.44^\circ} \text{ kVA}.$$

The power factor is

$$\text{pf} = \cos 38.44^\circ = 0.783 \text{ lagging}.$$

Finally, note that if we calculate the total complex power at the sending end of the line, after first calculating the magnitude of the line current, we can use this value to calculate V_L. That is,

$$\sqrt{3}\, V_L I_L = 3(206.41) \times 10^3$$
$$V_L = \frac{3(206.41) \times 10^3}{\sqrt{3}(577.35)} = 619.23 \text{ V}.$$

Instantaneous Power in Three-Phase Circuits

Although we are primarily interested in average, reactive, and complex power calculations, the computation of the total *instan-*

taneous power is also important. The total instantaneous power in a balanced three-phase circuit has an interesting property: It is invariant with time!

This can be shown as follows. Let the instantaneous line-to-neutral voltage v_{AN} be the reference and, as before, θ_ϕ be the phase angle $\theta_{vA} - \theta_{iA}$. Then, for a positive phase sequence, the instantaneous power in each phase is

$$p_A = v_{AN} i_{aA} = V_\phi I_\phi \cos \omega t \cos(\omega t - \theta_\phi),$$

$$p_B = v_{BN} i_{bB} = V_\phi I_\phi \cos(\omega t - 120°) \cos(\omega t - \theta_\phi - 120°),$$

and

$$p_C = v_{CN} i_{cC} = V_\phi I_\phi \cos(\omega t + 120°) \cos(\omega t - \theta_\phi + 120°),$$

where V_ϕ and I_ϕ represent the maximum values of the line-to-neutral voltage and line current, respectively. The instantaneous total power is the sum of the instantaneous phase powers, and this sum can be shown to reduce to $1.5 V_\phi I_\phi \cos \theta_\phi$; that is,

$$p_T = p_A + p_B + p_C = 1.5 V_\phi I_\phi \cos \theta_\phi.$$

We will leave this reduction for the reader (see Problem 13.39).

The fact that the total instantaneous power in a three-phase circuit is constant is an important property of three-phase circuits. It means that the torque developed at the shaft of a three-phase motor is constant, and this in turn means less vibration in machinery powered by three-phase motors.

Three-phase instantaneous power is constant

DRILL EXERCISES

13.10 The three-phase average power rating of the central processing unit (CPU) on a mainframe digital computer is 22,659 W. The three-phase line supplying the computer has a line voltage rating of 208 V (rms). The line current is 73.8 A (rms).

a) Calculate the total magnetizing reactive power absorbed by the CPU.
b) Calculate the power factor.

ANSWER: (a) 13,909.50 VAR; (b) 0.852 lagging.

13.11 The complex power associated with each phase of a balanced load is 384 + *j*288 kVA. The line voltage at the terminals of the load is 4160 V.

a) What is the magnitude of the line current feeding the load?
b) Given that the load is connected in delta and the impedance of each phase consists of a resistance in parallel with a reactance, calculate R and X.
c) Given that the load is wye-connected and the impedance of each phase consists of a resistance in series with a reactance, calculate R and X.

ANSWER: (a) 199.85 A; (b) $R = 45.07\ \Omega$, $X = 60.09\ \Omega$; (c) $R = 9.61\ \Omega$, $X = 7.21\ \Omega$.

13.12 A balanced bank of delta-connected capacitors is connected in parallel with the load described in Drill Exercise 13.11. The line voltage at the terminals of the load remains at 4160 V. The circuit is operating at a frequency of 60 Hz. The capacitors are adjusted so that the magnitude of the line current feeding the parallel combination of the load and capacitor bank is at its minimum.

a) What is the size of each capacitor in microfarads?

b) Repeat part (a), given that the capacitors are connected in a wye.

c) What is the magnitude of the line current?

ANSWER: (a) 44.14 μF; (b) 132.42 μF; (c) 159.88 A.

13.8 MEASUREMENT OF AVERAGE POWER IN THREE-PHASE CIRCUITS

The basic instrument used to measure power in three-phase circuits is the electrodynamometer wattmeter. It contains two coils. One coil, called the *current coil,* is stationary and is designed to carry a current that is proportional to the load current. The second coil, called the *potential coil,* is movable and carries a current that is proportional to the load voltage. The average deflection of the pointer that is attached to the movable coil is proportional to the product of the effective value of the current in the current coil, the effective value of the voltage impressed on the potential coil, and the cosine of the phase angle between this current and voltage. The direction in which the pointer deflects depends on the instantaneous polarity of the current-coil current and the potential-coil voltage. Therefore each coil has one terminal with a polarity mark—usually a plus sign—but sometimes the double polarity mark ± is used. The wattmeter deflects up-scale when (1) the polarity-marked terminal of the current coil is toward the source and (2) the polarity-marked terminal of the potential coil is connected to the same line in which the current coil has been inserted. The important features of the wattmeter are shown in Fig. 13.18.

The Two-Wattmeter Method

Before showing how we can use two electrodynamometer wattmeters to measure the total power delivered to a three-phase load, let us make an observation that clearly shows that only two wattmeters are needed. Consider a general network inside a "box" that is being energized by n conductors. The system is shown in Fig. 13.19. If we wish to measure, or calculate, the total power at the terminals of the box, we need to know $n - 1$ currents and voltages. This follows because if we choose one terminal as a reference, then there are only $n - 1$ independent

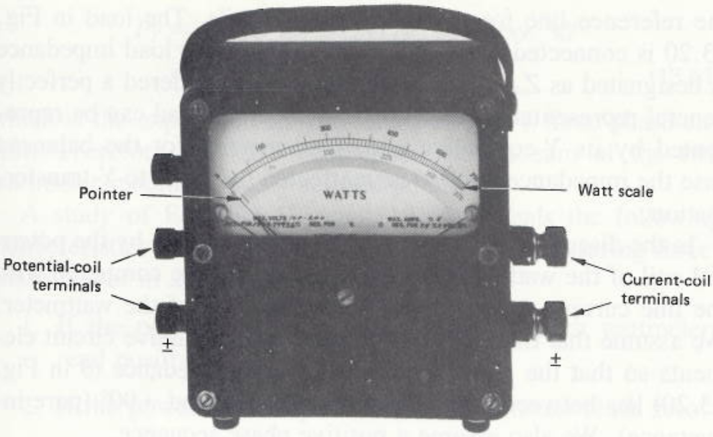


Figure 13.18 The important features of the electrodynamometer wattmeter.

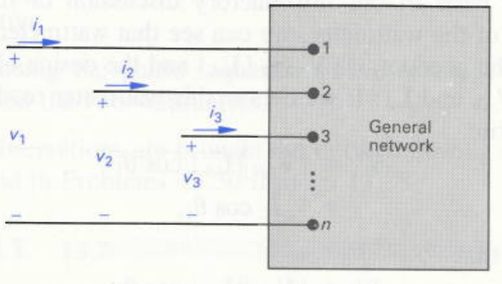


Figure 13.19 A circuit used to show that only $n - 1$ wattmeters are needed to measure the total power being delivered to the network.

voltages. Likewise, only $n - 1$ independent currents can exist in the n conductors entering the box. Thus the total power involves summing the products of $n - 1$ terms; that is, $p = v_1 i_1 + v_2 i_2 + \cdot \cdot \cdot + v_{n-1} i_{n-1}$.

Applying this general observation to a three-phase circuit, we can see that for a three-conductor circuit, whether balanced or not, we need only two wattmeters to measure the total power. For a four-conductor circuit, we need three wattmeters if the three-phase circuit is unbalanced, but only two wattmeters if it is balanced. We can conclude then that for any balanced three-phase circuit, we need only two wattmeters to measure the total power.

The two-wattmeter method of measuring the total power in a balanced three-phase circuit reduces to determining the magnitude and algebraic sign of the average power indicated by each wattmeter. We can describe the basic problem in terms of the circuit shown in Fig. 13.20, where the two wattmeters are indicated by the shaded boxes and labeled W_1 and W_2. The coil notations cc and pc stand for current coil and potential coil, respectively. We have elected to insert arbitrarily the current coils of the wattmeters in lines aA and cC. As a consequence, line bB is

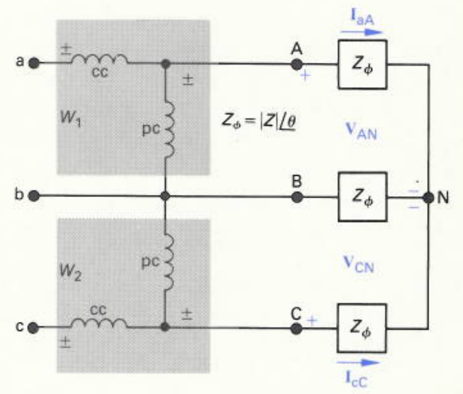


Figure 13.20 A circuit used to analyze the two-wattmeter method of measuring average power delivered to a balanced load.

the reference line for the two potential coils. The load in Fig. 13.20 is connected as a wye, and the per-phase load impedance is designated as $Z_\phi = |Z|\underline{/\theta}$. This can be considered a perfectly general representation since any Δ-connected load can be represented by its Y-equivalent and, furthermore, for the balanced case the impedance angle θ is unaffected by the Δ-to-Y transformation.

In the discussion that follows, the current drawn by the potential coil of the wattmeter is considered negligible compared with the line current measured by the current coil of the wattmeter. We assume that the loads can be modeled by passive circuit elements so that the phase angle of the load impedance (θ in Fig. 13.20) lies between −90° (pure capacitance) and +90° (pure inductance). We also assume a positive phase sequence.

On the basis of our introductory discussion of the average deflection of the wattmeter, we can see that wattmeter 1 will respond to the product of $|\mathbf{V}_{AB}|$, $|\mathbf{I}_{aA}|$ and the cosine of the angle between $\mathbf{V}_{AB}$ and $\mathbf{I}_{aA}$. If we denote this wattmeter reading as W_1, we can write

$$\begin{aligned} W_1 &= |\mathbf{V}_{AB}||\mathbf{I}_{aA}| \cos \theta_1 \\ &= V_L I_L \cos \theta_1. \end{aligned} \qquad \textbf{(13.55)}$$

It follows that

$$\begin{aligned} W_2 &= |\mathbf{V}_{CB}||\mathbf{I}_{cC}| \cos \theta_2 \\ &= V_L I_L \cos \theta_2. \end{aligned} \qquad \textbf{(13.56)}$$

In Eq. (13.55), θ_1 is the phase angle between $\mathbf{V}_{AB}$ and $\mathbf{I}_{aA}$ and in Eq. (13.56), θ_2 is the phase angle between $\mathbf{V}_{CB}$ and $\mathbf{I}_{cC}$.

To calculate W_1 and W_2, we express θ_1 and θ_2 in terms of the impedance angle θ, which is also the same as the phase angle between the phase voltage and phase current. For a positive phase sequence,

$$\theta_1 = \theta + 30° = \theta_\phi + 30° \qquad \textbf{(13.57)}$$

and

$$\theta_2 = \theta - 30° = \theta_\phi - 30°. \qquad \textbf{(13.58)}$$

The derivation of Eqs.13.57 and 13.58 is left as an exercise (see Problem 13.29). When we substitute Eqs. (13.57) and (13.58) into Eqs. (13.55) and (13.56) we get

$$W_1 = V_L I_L \cos (\theta + 30°) \qquad \textbf{(13.59)}$$

and

$$W_2 = V_L I_L \cos (\theta - 30°). \qquad \textbf{(13.60)}$$

To find the total power, we add W_1 and W_2; thus

$$P_T = W_1 + W_2 = 2V_L I_L \cos\theta_\phi \cos 30^\circ = \sqrt{3} V_L I_L \cos\theta_\phi, \quad (13.61)$$

which is the expression for the total power in a three-phase circuit. Therefore we have confirmed that the sum of the two wattmeter readings yields the total power.

A study of Eqs. (13.59) and (13.60) reveals the following characteristics of the two-wattmeter method of measuring three-phase power in a balanced circuit:

1. If the power factor is greater than 0.5, both wattmeters read positive.
2. If the power factor equals 0.5, one wattmeter reads zero.
3. If the power factor is less than 0.5, one wattmeter reads negative.
4. Reversing the phase sequence will interchange the readings on the two wattmeters.

These observations are brought out in the following numerical example and in Problems 13.30 through 13.38.

EXAMPLE 13.7

Two-wattmeter method calculations

Calculate the reading of each wattmeter in the circuit of Fig. 13.20 if the line-to-neutral voltage at the load is 120 V and (a) $Z_\phi = 8 + j6\ \Omega$; (b) $Z_\phi = 8 - j6\Omega$; (c) $Z_\phi = 5 + j5\sqrt{3}\Omega$; and (d) $Z_\phi = 10\underline{/-75^\circ}$. (e) Verify for parts (a) through (d) that the sum of the wattmeter readings equals the total power delivered to the load.

SOLUTION

a) $Z_\phi = 10\underline{/36.87^\circ}\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 120/10 = 12$ A.

$$W_1 = (120\sqrt{3})(12)\cos(36.87^\circ + 30^\circ) = 979.75 \text{ W},$$

$$W_2 = (120\sqrt{3})(12)\cos(36.87^\circ - 30^\circ) = 2476.25 \text{ W}.$$

b) $Z_\phi = 10\underline{/-36.87^\circ}\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 12$ A.

$$W_1 = (120\sqrt{3})(12)\cos(-36.87^\circ + 30^\circ) = 2476.25 \text{ W},$$

$$W_2 = (120\sqrt{3})(12)\cos(-36.87^\circ - 30^\circ) = 979.75 \text{ W}.$$

c) $Z_\phi = 5(1 + j\sqrt{3}) = 10\underline{/60^\circ}\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 12$ A.

$$W_1 = (120\sqrt{3})(12)\cos(60^\circ + 30^\circ) = 0,$$
$$W_2 = (120\sqrt{3})(12)\cos(60^\circ - 30^\circ)$$
$$= 2160 \text{ W}.$$

d) $Z_\phi = 10\underline{/-75^\circ}\ \Omega$, $V_L = 120\sqrt{3}$ V, $I_L = 12$ A.

$$W_1 = (120\sqrt{3})(12)\cos(-75^\circ + 30^\circ) = 1763.63 \text{ W},$$
$$W_2 = (120\sqrt{3})(12)\cos(-75^\circ - 30^\circ) = -645.53 \text{ W}.$$

e)
$$P_T(\text{a}) = 3(12)^2(8) = 3456 \text{ W},$$
$$W_1 + W_2 = 979.75 + 2476.25$$
$$= 3456 \text{ W};$$
$$P_T(\text{b}) = P_T(\text{a}) = 3456 \text{ W},$$
$$W_1 + W_2 = 2476.25 + 979.75$$
$$= 3456 \text{ W};$$
$$P_T(\text{c}) = 3(12)^2(5) = 2160 \text{ W},$$
$$W_1 + W_2 = 0 + 2160$$
$$= 2160 \text{ W};$$
$$P_T(\text{d}) = 3(12)^2(2.5882) = 1118.10 \text{ W},$$
$$W_1 + W_2 = 1763.63 - 645.53$$
$$= 1118.10 \text{ W}.$$

DRILL EXERCISES

13.13 The two-wattmeter method is used to measure the power at the load end of the line in Example 13.1. Calculate the reading of each wattmeter.

ANSWER: 197.29 W; 476.63 W.

13.14 The two-wattmeter method is used to measure the power at the sending end of the line in Example 13.3. Calculate the reading of each wattmeter.

ANSWER: 20,680.70 W; 8097.70 W.

13.15 The two wattmeters in Fig. 13.20 can be used to compute the total reactive power of the load.

a) Prove this statement by showing that $\sqrt{3}(W_2 - W_1) = \sqrt{3}\, V_L I_L \sin\theta_\phi$.

b) Compute the total reactive power from the wattmeter readings for each of the loads in Example 13.7. Check your computations by calculating the total reactive power directly from the given voltage and impedance.

ANSWER: (b) 2592 VAR, −2592 VAR, 3741.23 VAR, −4172.80 VAR.

SUMMARY

Our purpose has been to introduce you to the steady-state sinusoidal behavior of balanced three-phase circuits. When a three-phase circuit operates in a balanced mode, there are significant analytical shortcuts that can be used to calculate currents and voltages of interest. The key to these shortcuts is to reduce a given system to a single-phase equivalent circuit, a technique that relies on being able to make Δ-to-Y transformations at both the source end and load end of the circuit. Once the single-phase equivalent circuit has been derived, it is used to calculate the line current and the line-to-neutral voltages of interest. The current and the voltages obtained from the single-phase equivalent circuit can be translated into any other system current or voltage that is of interest. The translation from the single-phase equivalent circuit values to any other current or voltage in the circuit is based on the following observations.

1. In a balanced system, b- and c-phase currents and voltages are identical to the corresponding a-phase current and voltage except for a 120° shift in phase. In a positive sequence circuit, the b-phase quantity will lag the a-phase quantity by 120° and the c-phase quantity will lead the a-phase quantity by 120°. For a negative sequence circuit, phases b and c are interchanged with respect to phase a.
2. The set of line voltages is out of phase with the set of line-to-neutral voltages by ± 30°. The plus and minus sign corresponds to positive and negative sequence, respectively.
3. The magnitude of a line voltage is $\sqrt{3}$ times as large as the magnitude of a line-to-neutral voltage.
4. The set of line currents is out of phase with the set of phase currents in Δ-connected sources and loads by ∓ 30°. The minus and plus sign corresponds to positive and negative sequence, respectively.
5. The magnitude of a line current is $\sqrt{3}$ times as large as the magnitude of a phase current in the Δ-connected source or load.

Real, reactive, or complex power calculations can be made on either a per-phase basis or a total three-phase basis. The techniques for calculating real, reactive, or complex power on a per-phase basis are the same as those introduced in Chapter 12. The calculation of the total real, reactive, or complex power is based on using line current and line voltage, as expressed in Eqs. (13.37), (13.39), and (13.42).

PROBLEMS

All phasor voltages are stated in terms of the rms value.

13.1 For each set of voltages given below, state whether or not the voltages form a balanced three-phase set. If the set is a balanced set, state whether the phase sequence is positive or negative. If the set is not balanced, explain why.

a) $v_a = 294 \cos 377t$ V,
$v_b = 294 \cos (377t + 120°)$ V,
$v_c = 294 \cos (377t + 240°)$ V.

b) $v_a = 170 \sin 377t$ V,
$v_b = 170 \sin (377t - 120°)$ V,
$v_c = 170 \sin (377t + 120°)$ V.

c) $v_a = 400 \sin 377t$ V,
$v_b = -400 \cos (377t - 30°)$ V,
$v_c = 400 \cos (377t + 30°)$ V.

d) $v_a = 175 \cos (377t - 60°)$ V,
$v_b = 100\sqrt{3} \cos (377t + 60°)$ V,
$v_c = 175 \cos (377t - 180°)$ V.